This paper gives a simple introduction to the $SO(5)$ theory of high T_c superconductivity. Current status and relation to experiments are summarized.

The $SO(5)$ theory of high T_c superconductivity

Shou-Cheng Zhang[a]

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One of the most strikingly universal properties of high T_c superconductivity is the close proximity between the superconducting (SC) and the antiferromagnetic (AF) phases. The AF exchange coupling J is responsible for the AF phase at half-filling. The same couping could lead for formation of spin singlets, a prerequesite for superconductivity. On the other hand, while the origins of these two phenomena appear to be related, it is hard to imagine a greater difference in the physical properties between a insulator and a superconductor. Is it possible that behind the apparent difference, these two phases enjoy a deeper and fundamental unity?

Let us take the example of electricity and magnetism in vacuum, two of the best understood phenomena in physics. Before the 18th century, it is widely believed that these two phenomena are fundamentally distinct. However, the works of Faraday and Maxwell showed that they are in fact deeply related. In the theory of relativity, the electric field $\vec{E}$ and the magnetic field $\vec{B}$ are unified into a electromagnetic field tensor $F_{\mu\nu}$. The magnetic field $\vec{B}$ is nothing but the electric field $\vec{E}$ viewed from a rotated coordinate system in the four dimensional Minkowski space. Is it possible, that the two basic phases of the cuperates, AF and SC phases, are in fact related to each other by a simple rotation in some higher dimensional space?

A recently proposed theory of high T_c superconductivity unifies AF and the $d-$wave SC phases and treat them on equal footing[1]. The AF phase is described by a three dimensional order parameter N_α, the staggered magnetization. Therefore, it has spin 1, charge 0 and total momentum (π,π). On the other hand, a spin singlet d-wave SC phase is described by a complex order parameter Δ with two real components, which has spin 0, charge ± 2 and total momentum 0. The idea of the $SO(5)$ theory is to group these five components together into a object called superspin, $n_a = (Re\Delta, N_x, N_y, N_z, Im\Delta)$ and ask if there exists well-defined rotation operators which can transform AF into SC and vice versa. Such operators must have spin 1, charge ± 2 and total momentum (π,π) in order to patch up the difference between the AF and SC order parameters. These quantum numbers determine the form of the operator uniquely up to a form factor. One of them[2] is given by $\pi^\dagger = \sum_{\mathbf{k}} (\cos k_x - \cos k_y) c^\dagger_{\mathbf{k}+\pi,\uparrow} c^\dagger_{-\mathbf{k},\uparrow}$ Since this operator has spin 1, one can obviously define three of them $\pi^\dagger_\alpha$, where α is a spin index. This operator also has charge $+2$, one can therefore define its hermitian conjugate π_α which has charge -2. Remarkably, these operators rotate AF order parameter into the d-wave order parameter

$$[\pi^\dagger_\alpha, N_\beta] = i\delta_{\alpha\beta}\Delta^\dagger \tag{1}$$

and vice versa. Together with the total spin operators S_α and the total charge operator Q, the six π's can be identified with the 10 components of a antisymmetric tensor $L_{ab} = -L_{ba}$ in five dimensions, and they form the generators of the five dimensional rotation group, $SO(5)$. The $SO(5)$ group contains the familiar $SO(3)\times U(1)$ spin and charge symmetry as a subgroup.

We have thus apparently accomplished the task of unifying AF with SC: they are grouped into a five dimensional object and SC is nothing but AF viewed in some rotated coordinates and vice versa! This construction looks a bit similar to the unification of $\vec{E}$ and $\vec{B}$ by the Lorentz group. But so far this is only a mathematical fantasy, we haven't asked if Mother Nature approves the $SO(5)$ symmetry or not. In the high T_c problem, Mother Nature is very complicated, but we can check the $SO(5)$ symmetry within some simple models. One can easily check the $SO(5)$ symmetry by evaluating the commutator between the Hamiltonian with the π operators. Analytical[2] and numerical[3] works show that that the π operators are approximate eigen-operators of the Hubbard Hamiltonian, in the sense that

$$[\mathcal{H}, \pi^\dagger_\alpha] = \omega_0 \pi^\dagger_\alpha \tag{2}$$

where eigen-frequency ω_0 is of the order of J, and proportional to the number of holes. This relation reminds us of the commutation relation between the transverse spin components in a magnetic field and the Zeeman Hamiltonian. Therefore, the $SO(5)$ symmetry is broken explicitly by the chemical potential, but the pattern of explicit symmetry breaking is simple and familiar, and therefore easy to handle. In particular, the Casimir operator $\sum_{a<b} L_{ab}^2$ (a generalization of the total spin operator $\vec{S}^2$) of the $SO(5)$ group commutes with the Hamiltonian if (2) is valid. The approximate relation (2) is highly nontrivial. One could ask if a similar relation would exist for a modified π operator which rotates AF into s-wave SC order parameters. The answer is negative[2,3]. Therefore, there is only a approximate symmetry between AF and d-wave SC near half filling.

The ideas of the $SO(5)$ symmetry can now be used to construct a poor man's model of a high T_c superconductor. The reason that the high T_c problem appear to be hopelessly complicated is because fermion problems are hard to deal with. But the recent experimental discovery of the pseudogap[4] gives a ray of hope. At a temperature T_{MF} higher than the true T_c, the fermions already appear to be gapped. Better yet, these two temperature scales go in opposite direction as one approaches the AF phase from the SC side. This means that for low temperature properties in the underdoped regime, one can forget about the fermions and concentrate on the collective degrees of freedom, and map the whole problem into a "effective magnetic problem" involving the $SO(5)$ superspin.

We can define T_{MF} to be the temperature below which the superspin acquires a finite magnitude, but its orientation is still not fixed. Below this temperature, the fermionic degrees of freedom are quenched. One can always rescale the parameters so that the constraint $n_a^2 = 1$ is satisfied. We can define superspin vectors locally and end up with a quantum rotor model with some moment of inertia and gradient coupling. At half filling, where the chemical potential is defined to vanish, we assume that the $SO(5)$ symmetry is broken in such a way that AF phase is favored. This is similar to a magnetic probelm with "easy directions". The most important question is to ask what happens when the chemical potential deviates from zero.

The chemical potential μ couples to the charge, one of the symmetry generators in the $SO(5)$ theory. It does not couple to either AF or SC order parameters directly. Therefore, one's naive expectation is that the superspin direction is unaffected. However, a close inspection shows that the rotation generators are not independent of the order parameters. The simplest example of this kind of relation is the constraint between the total spin S_α and the Neel vector N_α of a antiferromagnet: $S_\alpha N_\alpha = 0$. Neel first derived this result by expressing S_α and N_α in terms of the sum and the difference of the sublattice magnetization. However, this orthogonality relation has a simple geometric interpretation. The $\vec{N}$ vector is confined to lie on a sphere. The $\vec{S}$ operator is a generator of the rotation of the $\vec{N}$ vector, therefore it has to lie in a plane tangent to the sphere. The Neel orthogonality relation has a extremely important physical consequence. If we apply a uniform magnetic field to a antiferromagnet, it induces a total spin along the field direction. The constraint relation immediately tells us that the Neel vector has to be perpendicular to the field direction. This phenomenon is called a spin-flop transition induced by a uniform magnetic field.

How does this picture generalize to higher component spins? In higher dimensions, the rotation generators can not be described as vectors, but can always be described by antisymmetric tensors L_{ab}. The index ab specify a plane tangent to a sphere traced out by the n_c vector, and the generalized orthogonality relation reads

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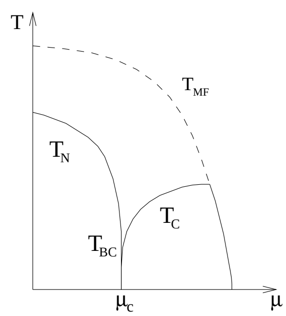
This equation can be proved by expressing the angular momentum operator in the usual fashion, $L_{ab} = n_a p_b - n_b p_a$, where p_a is the momentum conjugate of n_a, and substituting it back into (3). This simple equation expresses a geometric property of a hyper-sphere, but interpreted in the $SO(5)$ theory, it provides a powerful constraint between three most important physical quantities in the high T_c problem: doping (which is the convention of [1] is identified with L_{15}), AF (n_2, n_3, n_4) and SC (n_1, n_5). Since only the charge operator L_{15} couples to a external field, namely the chemical potential, the expectation values of all other generators vanish. The constraint equation (3) immediately tells us that at finite doping, the superspin has vanishing AF components and a finite SC component. Therefore, while the Neel constraint equation tells us about the physics of the spin flop, the $SO(5)$ constraint equation (3) tells us about the physics of the superspin flop, *i.e.* the transition from a AF phase to the SC phase induced by the chemical

potential. In the $SO(5)$ theory, the mysterious transition from AF to SC upon doping is explained by a simple geometric property of a sphere is five dimensions!

Once the analogy with the spin flop problem and the superspin flop problem is realized, one can simply borrow the spin-flop phase diagram to construct the phase diagram of the high T_c superconductors in the temperature and chemical potential plane, see figure 1. When the chemical potential μ is less than some critical value, say μ_c, the superspin prefers to lie in the AF direction. The chemical potential plays the role similar to the uniform magnetic field in the spin flop problem. Beyond the critical μ_c, the superspin flops from the AF direction to the SC plane. When temperature is raised at a constant chemical potential, the AF state undergoes a second order transition at T_N, while the SC state undergoes a second order transition at T_c. These two second order transition lines meet at a bicritical point T_{bc}. At this point, the $SO(5)$ symmetry becomes exact due to critical fluctuations. Since this point has the most thermal and quantum fluctuations in the entire phase diagram, both T_c and T_N are depressed near μ_c. The $SO(5)$ theory predicts that both second order lines merge into the first order lines tangentially, with a behavior close to a square root singularity. Because of the materials difficulty in the underdoped regime, it is experimentally unclear if T_c and T_N actually meet at a single point or are detached from each other. More experimental work in this direction is clearly desired to test this crucial prediction of the $SO(5)$ theory.



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So far the most direct evidence of the $SO(5)$ symmetry come from the resonant neutron scattering peaks in the $YBCO$ superconductors below T_c[6]. These resonances have spin 1, momentum (π, π), and resolution limited peaks at $41meV$, $33meV$ and $25meV$ for materials with $T_c = 92K$, $T_c = 67K$ and $T_c = 52K$ respectively. The resonance energy scales with T_c, but is not simply related to the size of the SC gap, since recent photoemission experiments show that the SC gap increases with decreasing doping[5].

These resonance peaks have a natural explanation within the $SO(5)$ theory. In the previous discussions, we argued that beyond a critical chemical potential μ_c, the superspin vector lies within the SC plane (n_1, n_5). However, this notion is a classical one, since the Heisenberg uncertainty relation does not allow the angle of the superspin to be sharply defined. In fact, there is zero point motion of the superspin into the AF directions (n_2, n_3, n_4). What are the appropriate coordinates describing this zero point motion? Let us recall the fundamental $SO(5)$ commutation relation (1), which tells us that the $\pi_\alpha^\dagger$ operator rotates AF into SC. In the SC state, we can approximate the right-hand-side of (1) by a c-number expectation value, and this equation can now be interpreted as the Heisenberg commutation relation between the canonical momentum p and coordinate q of a harmonic oscillator. The eigenfrequency of this oscillator can be simply read off from equation (2). This oscillator can be naturally identified with the resonant neutron peak observed in the $YBCO$ superconductors. It has momentum (π, π), spin 1 and a resonance energy which scales with the hole doping. Since the harmonic oscillator interpretation crucially depends on the SC order parameter having a finite expectation value, one would expect the mode to disappear

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A number of other experimental consequences of the $SO(5)$ are currently being worked out. The $SO(5)$ theory predicts that a SC vortex in the underdoped regime has a AF core[1,7]. Such a configuration is called a "meron" in field theory literature. In this configuration, the superspin lies within the SC plane far away from the vortex core, but it rotates from the SC plane into the AF sphere as the vortex core is approached in the radial direction. A vortex lattice with AF core could be detected as satellite peaks in the elastic neutron scattering experiment. It can also be detected by muon spin resonance inside the vortex core. Since the AF vector lies in a plane perpendicular to the applied field, this could give a distinct signature in the μSR experiment. The $SO(5)$ theory also predicts a charge doublet excitation in the AF state, which is the cousin of the spin triplet excitation in the SC state. In a AF state, the superspin vector lies within the AF sphere, however, its zero point motion leads to a fluctuation into the SC plane. Similar to the spin triplet resonance, the charge doublet resonance should only appear below the Neel temperature T_N. Although the phase diagram of the $SO(5)$ theory is qualitatively similar to many other alternative theories, it distinctively predicts a direct first order phase transition from AF to SC and a bi-critical point where both T_N and T_c merge. It is hard to access the AF/SC transition region experimentally because of sample inhomogeneities. This is certainly a major challenge which can hopefully be resolved experimentally in the near future.

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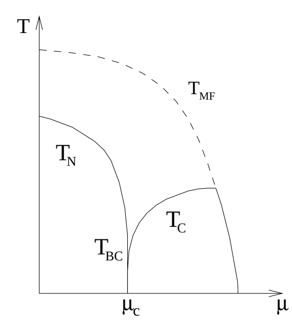

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